

Potentially $K_m - G$ -graphical Sequences: A Survey *

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Abstract

The set of all non-increasing nonnegative integers sequence $\pi = (d(v_1), d(v_2), \dots, d(v_n))$ is denoted by NS_n . A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π . The set of all graphic sequences in NS_n is denoted by GS_n . A graphical sequence π is potentially H -graphical if there is a realization of π containing H as a subgraph, while π is forcibly H -graphical if every realization of π contains H as a subgraph. Let K_k denote a complete graph on k vertices. Let $K_m - H$ be the graph obtained from K_m by removing the edges set $E(H)$ of the graph H (H is a subgraph of K_m). This paper summarizes briefly some recent results on potentially $K_m - G$ -graphic sequences and give a useful classification for determining $\sigma(H, n)$.

Key words: graph; degree sequence; potentially $K_m - G$ -graphic sequences

AMS Subject Classifications: 05C07, 05C35

1 Introduction

The set of all non-increasing nonnegative integer sequence $\pi = (d(v_1), d(v_2), \dots, d(v_n))$ is denoted by NS_n . A sequence $\pi \in NS_n$ is said to be graphic

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if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π . The set of all graphic sequences in NS_n is denoted by GS_n . A graphical sequence π is potentially H -graphical if there is a realization of π containing H as a subgraph, while π is forcibly H -graphical if every realization of π contains H as a subgraph. If π has a realization in which the $r+1$ vertices of largest degree induce a clique, then π is said to be potentially A_{r+1} -graphic. Let $\sigma(\pi) = d(v_1) + d(v_2) + \dots + d(v_n)$, and $[x]$ denote the largest integer less than or equal to x . We denote $G + H$ as the graph with $V(G + H) = V(G) \cup V(H)$ and $E(G + H) = E(G) \cup E(H) \cup \{xy : x \in V(G), y \in V(H)\}$. Let K_k , C_k , T_k , and P_k denote a complete graph on k vertices, a cycle on k vertices, a tree on $k+1$ vertices, and a path on $k+1$ vertices, respectively. Let F_k denote the friendship graph on $2k+1$ vertices, that is, the graph of k triangles intersecting in a single vertex. For $0 \leq r \leq t$, denote the generalized friendship graph on $kt - kr + r$ vertices by $F_{t,r,k}$, where $F_{t,r,k}$ is the graph of k copies of K_t meeting in a common r set. We use the symbol Z_4 to denote $K_4 - P_2$. Let $K_m - H$ be the graph obtained from K_m by removing the edges set $E(H)$ of the graph H (H is a subgraph of K_m).

Given a graph H , what is the maximum number of edges of a graph with n vertices not containing H as a subgraph? This number is denoted $ex(n, H)$, and is known as the Turán number. In terms of graphic sequences, the number $2ex(n, H) + 2$ is the minimum even integer l such that every n -term graphical sequence π with $\sigma(\pi) \geq l$ is forcibly H -graphical. Erdős, Jacobson and Lehel [13] first consider the following variant: determine the minimum even integer l such that every n -term graphical sequence π with $\sigma(\pi) \geq l$ is potentially H -graphical. We denote this minimum l by $\sigma(H, n)$. Erdős, Jacobson and Lehel [13] showed that $\sigma(K_k, n) \geq (k-2)(2n-k+1) + 2$ and conjectured that equality holds. They proved that if π does not contain zero terms, this conjecture is true for $k=3$, $n \geq 6$. The conjecture is confirmed in [19] and [43-46]. Li et al. [46] and Mubayi [55] also independently determined the values $\sigma(K_r, 2k)$ for any $k \geq 3$ and $n \geq k$. Li and Yin [51] further determined $\sigma(K_r, n)$ for $r \geq 7$ and $n \geq 2r+1$. The problem of determining $\sigma(K_r, n)$ is completely solved.

Gould, Jacobson and Lehel [19] also proved that $\sigma(pK_2, n) = (p-1)(2n-p) + 2$ for $p \geq 2$; $\sigma(C_4, n) = 2\lceil \frac{3n-1}{2} \rceil$ for $n \geq 4$. Lai [29] gave a lower bound of $\sigma(C_k, n)$ and proved that $\sigma(C_5, n) = 4n-4$ for $n \geq 5$ and $\sigma(C_6, n) = 4n-2$ for $n \geq 7$. Lai [32] proved that $\sigma(C_{2m+1}, n) = m(2n-m-1) + 2$, for $m \geq 2, n \geq 3m$; $\sigma(C_{2m+2}, n) = m(2n-m-1) + 4$, for $m \geq 2, n \geq 5m-2$. Li and Luo [41] gave a lower bound of $\sigma({}_3C_l, n)$ and determined $\sigma({}_3C_l, n)$, $4 \leq l \leq 6, n \geq l$. Li, Luo and Liu [42] determined $\sigma({}_3C_l, n)$ for $3 \leq l \leq 8$ and $n \geq l$. and $\sigma({}_3C_9, n)$ for $n \geq 12$. Li and Yin [48] determined $\sigma({}_3C_l, n)$ for n sufficiently large. Yin, Li and Chen [68] determined $\sigma({}_kC_l, n)$, $l \geq 7, 3 \leq k \leq l$. Chen and Yin [9] determined the values

$\sigma(W_5, n)$ for $n \geq 11$ where W_r is a wheel graph on r vertices. For $r \times s$ complete bipartite graph $K_{r,s}$, Gould, Jacobson and Lehel[19] determined $\sigma(K_{2,2}, n)$. Yin et al. [63,65,69,70] determined $\sigma(K_{r,s}, n)$ for $s \geq r \geq 2$ and sufficiently large n . For $r \times s \times t$ complete 3-partite graph $K_{r,s,t}$, Erdős, Jacobson and Lehel[13] determined $\sigma(K_{1,1,1}, n)$. Lai[30] determined $\sigma(K_{1,1,2}, n)$. Yin[58] and Lai[34] independently determined $\sigma(K_{1,1,3}, n)$. Chen[7] determined $\sigma(K_{1,1,t}, n)$ for $t \geq 3$, $n \geq 2\lceil \frac{(t+5)^2}{4} \rceil + 3$. Chen[5] determined $\sigma(K_{1,2,2}, n)$ for $5 \leq n \leq 8$ and $\sigma(K_{2,2,2}, n)$ for $n \geq 6$. Let K_s^t denote the complete t partite graph such that each partite set has exactly s vertices. Guantao Chen, Michael Ferrara, Ronald J. Gould, John R. Schmitt[11] showed that $\sigma(K_s^t, n) = \pi(K_{(t-2)s} + K_{s,s}, n)$ and obtained the exact value of $\sigma(K_j + K_{s,s}, n)$ for n sufficiently large. Consequently, they obtained the exact value of $\sigma(K_s^t, n)$ for n sufficiently large. For $n \geq 5$, Ferrara, Jacobson and Schmitt[17] determined $\sigma(F_k, n)$ where F_k denotes the graph of k triangles intersecting at exactly one common vertex. In[16], Ferrara, Gould and Schmitt determined a lower bound for $\sigma(K_s^t, n)$ where K_s^t denotes the complete multipartite graph with t partite sets each of size s , and proved equality in the case $s = 2$. They also provided a graph theoretic proof of the value of $\sigma(K_t, n)$. Michael J. Ferrara[15] determined $\sigma(H, n)$ for the graph $H = K_{m_1} \cup K_{m_2} \cup \dots \cup K_{m_k}$ where n is sufficiently large integer. Ferrara, M., Jacobson, M., Schmitt, J. and Siggers M.[18] determined $\sigma(K_{s,t}, m, n)$, $\sigma(P_t, m, n)$ and $\sigma(C_{2t}, m, n)$ where $\sigma(H, m, n)$ is the minimum integer k such that every bigraphic pair $S = (A, B)$ with $|A| = m$, $|B| = n$ and $\sigma(S) \geq k$ is potentially H -bigraphic. For an arbitrarily chosen H , Schmitt, J.R. and Ferrara, M.[56] gave a good lower bound of $\sigma(H, n)$. Yin and Li[67] determined $\sigma(K_{r_1, r_2, \dots, r_l, r, s}, n)$ for sufficiently large n . Moreover, Yin, Chen and Schmitt[62] determined $\sigma(F_{t,r,k}, n)$ for $k \geq 2$, $t \geq 3$, $1 \leq r \leq t-2$ and sufficiently large, where $F_{t,r,k}$ denotes the graph of k copies of K_t meeting in a common r set. Gupta, Joshi and Tripathi [20] gave a necessary and sufficient condition for the existence of a tree of order n with a given degree set. Yin [59] gave a new necessary and sufficient condition for π to be potentially K_{r+1} -graphic. Jiong-sheng Li and Jianhua Yin [50] gave a survey on graphical sequences.

2 Potentially $K_m - G$ -graphical Sequences

Let H be a graph with m vertices, then $H = K_m - (K_m - H)$. Let $G = K_m - H$, then $\sigma(H, n) = \sigma(K_m - G, n)$. If the Problem 1 - 5 in the Open Problems section be solved, then the problem of determining $\sigma(H, n)$ is completely solved. We think the Problem 3, 4 is a useful classification for determining $\sigma(K_m - G, n)$.

Gould, Jacobson and Lehel[19] pointed out that it would be nice to see

where in the range for $3n - 2$ to $4n - 4$, the value $\sigma(K_4 - e, n)$ lies. Later, Lai[30] proved that

Theorem 1. For $n = 4, 5$ and $n \geq 7$

$$\sigma(K_4 - e, n) = \begin{cases} 3n - 1 & \text{if } n \text{ is odd} \\ 3n - 2 & \text{if } n \text{ is even.} \end{cases}$$

For $n = 6$, S is a 6-term graphical sequence with $\sigma(S) \geq 16$, then either there is a realization of S containing $K_4 - e$ or $S = (3^6)$. (Thus $\sigma(K_4 - e, 6) = 20$).

Huang[26] gave a lower bound of $\sigma(K_m - e, n)$. Yin, Li and Mao[71] and Huang[27] independently determined the values $\sigma(K_5 - e, n)$ as following.

Theorem 2. If $n \geq 5$, then

$$\sigma(K_5 - e, n) = \begin{cases} 5n - 7, & \text{if } n \text{ is odd} \\ 5n - 6, & \text{if } n \text{ is even} \end{cases}$$

Lai[35-36] determined $\sigma(K_5 - C_4, n)$, $\sigma(K_5 - P_3, n)$ and $\sigma(K_5 - P_4, n)$.

Theorem 3. For $n \geq 5$, $\sigma(K_5 - C_4, n) = \sigma(K_5 - P_3, n) = \sigma(K_5 - P_4, n) = 4n - 4$.

Yin and Li[66] gave a good method of determining the values $\sigma(K_{r+1} - e, n)$ (In fact, Yin and Li[66] also determining the values $\sigma(K_{r+1} - ke, n)$ for $r \geq 2$ and $n \geq 3r^2 - r - 1$).

Theorem 4. Let $n \geq r + 1$ and $\pi = (d_1, d_2, \dots, d_n) \in GS_n$ with $d_{r+1} \geq r$. If $d_i \geq 2r - i$ for $i = 1, 2, \dots, r - 1$, then π is potentially A_{r+1} -graphic.

Theorem 5. Let $n \geq 2r + 2$ and $\pi = (d_1, d_2, \dots, d_n) \in GS_n$ with $d_{r+1} \geq r$. If $d_{2r+2} \geq r - 1$, then π is potentially A_{r+1} -graphic.

Theorem 6. Let $n \geq r + 1$ and $\pi = (d_1, d_2, \dots, d_n) \in GS_n$ with $d_{r+1} \geq r - 1$. If $d_i \geq 2r - i$ for $i = 1, 2, \dots, r - 1$, then π is potentially $K_{r+1} - e$ -graphic.

Theorem 7. Let $n \geq 2r + 2$ and $\pi = (d_1, d_2, \dots, d_n) \in GS_n$ with $d_{r-1} \geq r$. If $d_{2r+2} \geq r - 1$, then π is potentially $K_{r+1} - e$ -graphic.

Theorem 8. If $r \geq 2$ and $n \geq 3r^2 - r - 1$, then

$$\sigma(K_{r+1} - ke, n) = \begin{cases} (r - 1)(2n - r) - (n - r) + 1, & \text{if } n - r \text{ is odd} \\ (r - 1)(2n - r) - (n - r) + 2, & \text{if } n - r \text{ is even} \end{cases}$$

After reading[66], Yin[72] determined the values $\sigma(K_{r+1} - K_3, n)$ for $r \geq 3, n \geq 3r + 5$.

Theorem 9. If $r \geq 3$ and $n \geq 3r + 5$, then $\sigma(K_{r+1} - K_3, n) = (r - 1)(2n - r) - 2(n - r) + 2$.

Determining $\sigma(K_{r+1} - H, n)$, where H is a tree on 4 vertices is more useful than a cycle on 4 vertices (for example, $C_4 \not\subset C_i$, but $P_3 \subset C_i$ for $i \geq 5$). So, after reading [66] and [72], Lai and Hu [38] determined $\sigma(K_{r+1} - H, n)$ for $n \geq 4r + 10, r \geq 3, r + 1 \geq k \geq 4$ and H be a graph on k vertices which containing a tree on 4 vertices but not containing a cycle on 3 vertices and $\sigma(K_{r+1} - P_2, n)$ for $n \geq 4r + 8, r \geq 3$.

Theorem 10. If $r \geq 3$ and $n \geq 4r + 8$, then $\sigma(K_{r+1} - P_2, n) = (r - 1)(2n - r) - 2(n - r) + 2$.

Theorem 11. If $r \geq 3, r + 1 \geq k \geq 4$ and $n \geq 4r + 10$, then $\sigma(K_{r+1} - H, n) = (r - 1)(2n - r) - 2(n - r)$, where H is a graph on k vertices which contains a tree on 4 vertices but not contains a cycle on 3 vertices.

There are a number of graphs on k vertices which containing a tree on 4 vertices but not containing a cycle on 3 vertices (for example, the cycle on k vertices, the tree on k vertices, and the complete 2-partite graph on k vertices, etc).

Lai and Sun [39] determined $\sigma(K_{r+1} - (kP_2 \cup tK_2), n)$ for $n \geq 4r + 10, r + 1 \geq 3k + 2t, k + t \geq 2, k \geq 1, t \geq 0$.

Theorem 12. If $n \geq 4r + 10, r + 1 \geq 3k + 2t, k + t \geq 2, k \geq 1, t \geq 0$, then $\sigma(K_{r+1} - (kP_2 \cup tK_2), n) = (r - 1)(2n - r) - 2(n - r)$.

To now, the problem of determining $\sigma(K_{r+1} - H, n)$ for H not containing a cycle on 3 vertices and sufficiently large n has been solved.

Lai [37] determined $\sigma(K_{r+1} - Z, n)$ for $n \geq 5r + 19, r + 1 \geq k \geq 5, j \geq 5$ and Z is a graph on k vertices and j edges which contains a graph Z_4 but not contain a cycle on 4 vertices. In the same paper, the author also determined the values of $\sigma(K_{r+1} - Z_4, n)$, $\sigma(K_{r+1} - (K_4 - e), n)$, $\sigma(K_{r+1} - K_4, n)$ for $n \geq 5r + 16, r \geq 4$.

Theorem 13. If $r \geq 4$ and $n \geq 5r + 16$, then

$$\sigma(K_{r+1} - K_4, n) = \sigma(K_{r+1} - (K_4 - e), n) =$$

$$\sigma(K_{r+1} - Z_4, n) = \begin{cases} (r - 1)(2n - r) - 3(n - r) + 1, & \text{if } n - r \text{ is odd} \\ (r - 1)(2n - r) - 3(n - r) + 2, & \text{if } n - r \text{ is even} \end{cases}$$

Theorem 14. If $n \geq 5r + 19, r + 1 \geq k \geq 5$, and $j \geq 5$, then

$$\sigma(K_{r+1} - Z, n) = \begin{cases} (r - 1)(2n - r) - 3(n - r) - 1, & \text{if } n - r \text{ is odd} \\ (r - 1)(2n - r) - 3(n - r) - 2, & \text{if } n - r \text{ is even} \end{cases}$$

where Z is a graph on k vertices and j edges which contains a graph Z_4 but not contains a cycle on 4 vertices.

There are a number of graphs on k vertices and j edges which contains a graph Z_4 but not contains a cycle on 4 vertices. (for example, the graph obtained by $C_3, C_{i_1}, C_{i_2}, \dots, C_{i_p}$ intersecting in a single vertex ($i_j \neq 4, j = 1, 2, 3, \dots, p$) (if $i_j = 3, j = 1, 2, 3, \dots, p$, then this graph is the friendship graph F_{p+1}), the graph obtained by $C_3, P_{i_1}, P_{i_2}, \dots, P_{i_p}$ intersecting in a single vertex, ($i_1 \geq 1$), the graph obtained by $C_3, P_{i_1}, C_{i_2}, \dots, C_{i_p}$ ($i_j \neq 4, j = 2, 3, \dots, p, i_1 \geq 1$) intersecting in a single vertex, etc)

Lai and Yan[40] proved that

Theorem 15. If $n \geq 5r + 18, r + 1 \geq k \geq 7$, and $j \geq 6$, then

$$\sigma(K_{r+1} - U, n) = \begin{cases} (r-1)(2n-r) - 3(n-r) - 1, \\ \text{if } n-r \text{ is odd} \\ (r-1)(2n-r) - 3(n-r), \\ \text{if } n-r \text{ is even} \end{cases}$$

where U is a graph on k vertices and j edges which contains a graph $(K_3 \cup P_3)$ but not contains a cycle on 4 vertices and not contains Z_4 .

There are a number of graphs on k vertices and j edges which contains a graph $(K_3 \cup P_3)$ but not contains a cycle on 4 vertices and not contains Z_4 . (for example, $C_3 \cup C_{i_1} \cup C_{i_2} \cup \dots \cup C_{i_p}$ ($i_j \neq 4, j = 2, 3, \dots, p, i_1 \geq 5$), $C_3 \cup P_{i_1} \cup P_{i_2} \cup \dots \cup P_{i_p}$ ($i_1 \geq 3$), $C_3 \cup P_{i_1} \cup C_{i_2} \cup \dots \cup C_{i_p}$ ($i_j \neq 4, j = 2, 3, \dots, p, i_1 \geq 3$), etc)

A harder question is to characterize the potentially H -graphic sequences without zero terms. Luo [53] characterized the potentially C_k -graphic sequences for each $k = 3, 4, 5$.

Theorem 16. Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 3$. Then π is potentially C_3 -graphic if and only if $d_3 \geq 2$ except for 2 case: $\pi = (2^4)$ and $\pi = (2^5)$.

Theorem 17. Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 4$. Then π is potentially C_4 -graphic if and only if the following conditions hold:

- (1) $d_4 \geq 2$. (2) $d_1 = n - 1$ implies $d_2 \geq 3$
- (3) If $n = 5, 6$, then $\pi \neq (2^n)$.

Theorem 18. Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially C_5 -graphic if and only if the following conditions hold:

- (1) $d_5 \geq 2$. (2) For $i = 1, 2, d_1 = n - i$ implies $d_{4-i} \geq 3$
- (3) If $\pi = (d_1, d_2, 2^k, 1^{n-k-2})$, then $d_1 + d_2 \leq n + k - 2$.

Chen [2] characterized the potentially C_k -graphic sequences for each $k = 6$.

Theorem 19. Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 6$. Then π is potentially C_6 -graphic if and only if the following conditions hold:

- (1) $d_6 \geq 2$.
- (2) If $n = 7, 8$, then $\pi \neq (2^n)$.
- (3) For $i = 1, 2, 3$, $d_1 = n - i$ implies $d_{5-i} \geq 3$.
- (4) If $\pi = (d_1, d_2, 2^k, 1^{n-k-2})$, then $d_1 + d_2 \leq n + k - 2$; if $\pi = (d_1, d_2, 3, 2^k, 1^{n-k-3})$, then $d_1 + d_2 \leq n + k$; if $\pi = (d_1, d_2, 3, 3, 2^k, 1^{n-k-4})$, then $d_1 + d_2 \leq n + k + 2$.

Yin, Chen and Chen [60] characterized the potentially ${}_k C_l$ -graphic sequences for each $k = 3, 4 \leq l \leq 5$ and $k = 4, l = 5$.

Theorem 20. Let $\pi = (d_1, d_2, \dots, d_n) \in GS_n$ be a potentially C_4 -graphic sequence. Then π is potentially ${}_3 C_4$ -graphic if and only if π satisfies one of the following conditions:

- (1) $d_2 \geq 3$ and $\pi \neq (3^2, 2^4)$;
- (2) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $2 \leq d_1 \leq 3$ and $k \geq 6$, and $\pi \neq (2^8)$ and (2^9) ;
- (3) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $4 \leq d_1 \leq n - 2$ and $k \geq 5$, and $\pi \neq (4, 2^6)$ and $(4, 2^7)$.

Theorem 21. Let $\pi = (d_1, d_2, \dots, d_n) \in GS_n$ be a potentially C_5 -graphic sequence. Then π is potentially ${}_3 C_5$ -graphic if and only if π satisfies one of the following conditions:

- (1) $d_2 \geq 3$ and $\pi \neq (3^2, 2^4)$ and $(3^2, 2^5)$;
- (2) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $2 \leq d_1 \leq 3$ and $k \geq 11$, and $\pi \neq (2^{13})$ and (2^{14}) ;
- (3) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $4 \leq d_1 \leq 5$ and $k \geq 10$, and $\pi \neq (4, 2^{11})$ and $(4, 2^{12})$.
- (4) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $6 \leq d_1 \leq n - 4$ and $k \geq 9$, and $\pi \neq (4, 2^{10})$ and $(4, 2^{11})$.

Theorem 22. Let $\pi = (d_1, d_2, \dots, d_n) \in GS_n$ be a potentially C_5 -graphic sequence. Then π is potentially ${}_4 C_5$ -graphic if and only if π satisfies one of the following conditions:

- (1) $d_2 \geq 3$;
- (2) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $2 \leq d_1 \leq 3$ and $k \geq 8$, and $\pi \neq (2^{10})$ and (2^{11}) ;
- (3) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $4 \leq d_1 \leq n - 4$ and $k \geq 7$, and $\pi \neq (4, 2^8)$ and $(4, 2^9)$.

Chen, Yin and Fan [10] characterized the potentially ${}_k C_l$ -graphic sequences for each $3 \leq k \leq 5, l = 6$.

Theorem 23. Let $\pi = (d_1, d_2, \dots, d_n) \in GS_n$, $n \geq 6$, and $\pi \neq (3^2, 2^{10}), (2^{19}), (2^{20}), (4, 2^{17}), (4, 2^{18}), (6, 2^{16}), (6, 2^{17}), (8, 2^{15}), (8, 2^{16})$. Then π is potentially ${}_3 C_6$ -graphic if and only if π be a potentially C_6 -graphic sequence, and π satisfies one of the following conditions:

- (1) $d_3 \geq 3$, and if $d_1 = d_3 = 3, d_4 = 2$, then $d_{10} = 2$;
- (2) $d_2 \geq 4, d_3 = 2, d_7 = 2$;

(3) $d_2 = 3, d_3 = 2$, and if $4 \geq d_1 \geq 3$, then $d_{10} = 2$, and if $n-4 \geq d_1 \geq 5$, then $d_9 = 2$.

(4) $d_2 = 2$, and if $3 \geq d_1 \geq 2$, then $d_{18} = 2$, and if $5 \geq d_1 \geq 4$, then $d_{17} = 2$, and if $7 \geq d_1 \geq 6$, then $d_{16} = 2$, and if $n-7 \geq d_1 \geq 8$, then $d_{15} = 2$.

Theorem 24. Let $\pi = (d_1, d_2, \dots, d_n) \in GS_n$, $n \geq 6$, and $\pi \neq (2^{16}), (2^{17}), (4, 2^{14}), (4, 2^{15}), (6, 2^{13}), (6, 2^{14})$. Then π is potentially ${}_4C_6$ -graphic if and only if π be a potentially C_6 -graphic sequence, and π satisfies one of the following conditions:

(1) $d_3 \geq 3$, and if $d_1 = d_3 = 3, d_4 = 2$, then $d_{10} = 2$;

(2) $d_2 \geq 4, d_3 = 2, d_7 = 2$;

(3) $d_2 = 3, d_3 = 2$, and if $4 \geq d_1 \geq 3$, then $d_{10} = 2$, and if $n-4 \geq d_1 \geq 5$, then $d_9 = 2$.

(4) $d_2 = 2$, and if $3 \geq d_1 \geq 2$, then $d_{15} = 2$, and if $5 \geq d_1 \geq 4$, then $d_{14} = 2$, and if $n-7 \geq d_1 \geq 6$, then $d_{13} = 2$.

Theorem 25. Let $\pi = (d_1, d_2, \dots, d_n) \in GS_n$, $n \geq 6$, and $\pi \neq (2^{12}), (2^{13}), (4, 2^{10}), (4, 2^{11})$. Then π is potentially ${}_5C_6$ -graphic if and only if π be a potentially C_6 -graphic sequence, and π satisfies one of the following conditions:

(1) $d_2 \geq 3$;

(2) $3 \geq d_1 \geq 2, d_2 = 2, d_{11} = 2$;

(3) $n-6 \geq d_1 \geq 4, d_2 = 2, d_{10} = 2$.

Luo and Warner [54] characterized the potentially K_4 -graphic sequences.

Theorem 26. Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence without zero terms and with $d_4 \geq 3$ and $n \geq 4$. Then π is potentially K_4 -graphic if and only if $d_4 \geq 3$ and $\pi \neq (n-1, 3^s, !^{n-s-1})$ for each $s = 4, 5$ except the following sequences:

$n = 5 : (4, 3^4), (3^4, 2)$;

$n = 6 : (4^6), (4^2, 3^4), (4, 3^4, 2), (3^6), (3^5, 1), (3^4, 2^2)$;

$n = 7 : (4^7), (4^3, 3^4), (4, 3^6), (4, 3^5, 1), (3^6, 2), 3^5, 2, 1)$;

$n = 8 : (3^7, 1), (3^6, 1^2)$.

Eschen and Niu [14] and Lai[31] independently characterized the potentially $K_4 - e$ -graphic sequences.

Theorem 27. Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 4$. Then π is potentially $(K_4 - e)$ -graphic if and only if the following conditions hold:

(1) $d_2 \geq 3$. (2) $d_4 \geq 2$

(3) If $n = 5, 6$, then $\pi \neq (3^2, 2^{n-2})$ and $\pi \neq (3^6)$.

Yin and Yin [73] characterized the potentially $K_5 - e$ and K_6 -graphic sequences.

Theorem 28 Let $n \geq 5$ and $\pi = (d_1, d_2, \dots, d_n) \in NS_n$ be a positive graphic sequence with $d_3 \geq 4$ and $d_5 \geq 3$. Then π is potentially

$K_5 - e$ -graphic if and only if π is not one of the following sequences:
 $(n-1, 4^6, 1^{n-7}), (n-1, 4^2, 3^4, 1^{n-7}), (n-1, 4^2, 3^3, 1^{n-6});$

$n = 6: (4^6), (4^4, 3^2), (4^3, 3^2, 2);$

$n = 7: (4^3, 3^4), (5^2, 4, 3^4), (4^7), (4^5, 3^2), (5, 4^3, 3^3), (5^2, 4^5), (5, 4^5, 3),$
 $(4^3, 3^2, 2^2), (4^4, 3^2, 2), (5, 4^2, 3^3, 2), (4^6, 2), (4^3, 3^3, 1);$

$n = 8: (5^8), (4^8), (5^2, 4^6), (6, 4^7), (4^4, 3^4), (5, 4^2, 3^5), (4^6, 3^2), (5, 4^6, 3),$
 $(4^3, 3^4, 2), (4^7, 2), (4^4, 3^3, 1), (5, 4^2, 3^4, 1), (4^3, 3^3, 2, 1), (4^6, 3, 1), (5, 4^6, 1);$

$n = 9: (4^9), (4^3, 3^5, 1), (4^8, 2), (4^7, 3, 1), (5, 4^7, 1), (4^3, 3^4, 1^2), (4^7, 1^2);$

$n = 10: (4^8, 1^2).$

Theorem 29 Let $n \geq 18$ and $\pi = (d_1, d_2, \dots, d_n) \in NS_n$ be a positive graphic sequence with $d_6 \geq 5$. Then π is potentially A_6 -graphic if and only if $\pi_6 \notin \{(2), (2^2), (3, 1), (3^2), (3, 2, 1), (3^2, 2), (3^3, 1), (3^2, 1^2)\}$.

Yin and Chen [61] characterized the potentially $K_{r,s}$ -graphic sequences for $r = 2, s = 3$ and $r = 2, s = 4$.

Theorem 30 Let $n \geq 5$ and $\pi = (d_1, d_2, \dots, d_n) \in GS_n$. Then π is potentially $K_{2,3}$ -graphic if and only if π satisfies the following conditions:

(1) $d_2 \geq 3$ and $d_5 \geq 2$;

(2) If $d_1 = n-1$ and $d_2 = 3$, then $d_5 = 3$;

(3) $\pi \neq (3^2, 2^4), (3^2, 2^5), (4^3, 2^3), (n-1, 3^5, 1^{n-6})$ and $(n-1, 3^6, 1^{n-7})$.

Theorem 31 Let $n \geq 6$ and $\pi = (d_1, d_2, \dots, d_n) \in GS_n$. Then π is potentially $K_{2,4}$ -graphic if and only if π satisfies the following conditions:

(1) $d_2 \geq 4$ and $d_6 \geq 2$;

(2) If $d_1 = n-1$ and $d_2 = 4$, then $d_3 = 4$ and $d_6 \geq 3$;

(3) $\pi \neq (4^3, 2^4), (4^2, 2^5), (4^2, 2^6), (5^2, 4, 2^4), (5^3, 3, 2^3), (6, 5^2, 2^5), (5^3, 2^4, 1),$
 $(6^3, 2^6), (n-1, 4^2, 3^4, 1^{n-7}), (n-1, 4^2, 3^5, 1^{n-8}), (n-2, 4^2, 2^3, 1^{n-6}),$ and
 $(n-2, 4^3, 2^2, 1^{n-6}).$

Chen [3] characterized the potentially $K_5 - 2K_2$ -graphic sequences for $5 \leq n \leq 8$. Hu and Lai [23] characterized the potentially $K_5 - P_3, K_5 - A_3, K_5 - K_3, K_5 - K_{1,3}$ and $K_5 - 2K_2$ -graphic sequences where A_3 is $P_2 \cup K_2$.

Theorem 32 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - P_3$ -graphic if and only if the following conditions hold:

(1) $d_1 \geq 4, d_3 \geq 3$ and $d_5 \geq 2$.

(2) $\pi \neq (4, 3^2, 2^3), (4, 3^2, 2^4)$ and $(4, 3^6)$.

Theorem 33 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - A_3$ -graphic if and only if the following conditions hold:

(1) $d_4 \geq 3$ and $d_5 \geq 2$.

(2) $\pi \neq (n-1, 3^3, 2^{n-k}, 1^{k-4})$ where $n \geq 6$ and $k = 4, 5, \dots, n-2, n$ and k have the same parity.

(3) $\pi \neq (3^4, 2^2), (3^6), (3^4, 2^3), (3^6, 2), (4, 3^6), (3^7, 1), (3^8), (n-1, 3^5, 1^{n-6})$ and $(n-1, 3^6, 1^{n-7})$.

Theorem 34 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - K_3$ -graphic if and only if the following conditions hold:

- (1) $d_2 \geq 4$ and $d_5 \geq 2$.
- (2) $\pi \neq (4^2, 2^4), (4^2, 2^5), (4^3, 2^3)$ and (4^6) .

Theorem 35 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - K_{1,3}$ -graphic if and only if the following conditions hold:

- (1) $d_1 \geq 4$ and $d_4 \geq 3$.
- (2) $\pi \neq (4, 3^4, 2), (4^6), (4^2, 3^4), (4, 3^6), (4^7), (4, 3^5, 1), (n-1, 3^4, 1^{n-5})$ and $(n-1, 3^5, 1^{n-6})$.

Theorem 36 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - 2K_2$ -graphic if and only if the following conditions hold:

- (1) $d_1 \geq 4$ and $d_5 \geq 3$;
- (2)

$$\pi \neq \begin{cases} (n-i, n-j, 3^{n-i-j-2k}, 2^{2k}, 1^{i+j-2}) \\ \quad n-i-j \text{ is even;} \\ (n-i, n-j, 3^{n-i-j-2k-1}, 2^{2k+1}, 1^{i+j-2}) \\ \quad n-i-j \text{ is odd.} \end{cases}$$

where $1 \leq j \leq n-5$ and $0 \leq k \leq \lfloor \frac{n-j-i-4}{2} \rfloor$.

- (3) $\pi \neq (4^2, 3^4), (4, 3^4, 2), (5, 4, 3^5), (5, 3^5, 2), (4^7), (4^3, 3^4), (4^2, 3^4, 2), (4, 3^6), (4, 3^5, 1), (4, 3^4, 2^2), (5, 3^7), (5, 3^6, 1), (4^8), (4^2, 3^6), (4^2, 3^5, 1), (4, 3^6, 2), (4, 3^5, 2, 1), (4, 3^7, 1), (4, 3^6, 1^2), (n-1, 3^5, 1^{n-6})$ and $(n-1, 3^6, 1^{n-7})$.

Hu and Lai [21] characterized the potentially $K_5 - C_4$ -graphic sequences.

Theorem 37 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $(K_5 - C_4)$ -graphic if and only if the following conditions hold:

- (1) $d_1 \geq 4$.
- (2) $d_5 \geq 2$.
- (3) $\pi \neq ((n-2)^2, 2^{n-2})$ for $n \geq 6$, where the symbol x^y stands for y consecutive terms x .
- (4) $\pi \neq (n-k, k+i, 2^i, 1^{n-i-2})$ where $i = 3, 4, \dots, n-2k$ and $k = 1, 2, \dots, \lfloor \frac{n-1}{2} \rfloor - 1$.
- (5) If $n = 6$, then $\pi \neq (4, 2^5)$.
- (6) If $n = 7$, then $\pi \neq (4, 2^6)$.

Hu and Lai [22] characterized the potentially $K_5 - Z_4$ -graphic sequences.

Theorem 38 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $(K_5 - Z_4)$ -graphic if and only if the following conditions hold:

- (1) $d_1 \geq 4, d_2 \geq 3$ and $d_4 \geq 2$.

Hu, Lai and Wang[25] characterized the potentially $K_5 - P_4$ and $K_5 - Y_4$ -graphic sequences where Y_4 is a tree on 5 vertices and 3 leaves.

Theorem 39 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - P_4$ -graphic if and only if the following conditions hold:

- (1) $d_2 \geq 3$.
- (2) $d_5 \geq 2$.
- (3) $\pi \neq (n-1, k, 2^t, 1^{n-2-t})$ where $n \geq 5$, $k, t = 3, 4, \dots, n-2$, and, k and t have different parities.
- (4) For $n \geq 5$, $\pi \neq (n-k, k+i, 2^i, 1^{n-i-2})$ where $i = 3, 4, \dots, n-2k$ and $k = 1, 2, \dots, \lfloor \frac{n-1}{2} \rfloor - 1$.
- (5) If $n = 6, 7$, then $\pi \neq (3^2, 2^{n-2})$.

Theorem 40 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - Y_4$ -graphic if and only if the following conditions hold:

- (1) $d_3 \geq 3$.
- (2) $d_4 \geq 2$.
- (3) $\pi \neq (3^6)$.

Hu and Lai [24] characterized the potentially $K_{3,3}$ and $K_6 - C_6$ -graphic sequences.

Theorem 41 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 6$. Then π is potentially $K_{3,3}$ -graphic if and only if the following conditions hold:

- (1) $d_6 \geq 3$;
- (2) For $i = 1, 2$, $d_1 = n - i$ implies $d_{4-i} \geq 4$;
- (3) $d_2 = n - 1$ implies $d_3 \geq 5$ or $d_6 \geq 4$;
- (4) $d_1 + d_2 = 2n - i$ and $d_{n-i+3} = 1$ ($3 \leq i \leq n-4$) implies $d_3 \geq 5$ or $d_6 \geq 4$;
- (5) $d_1 + d_2 = 2n - i$ and $d_{n-i+4} = 1$ ($4 \leq i \leq n-3$) implies $d_3 \geq 4$;
- (6) $\pi = (d_1, d_2, 3^4, 2^t, 1^{n-6-t})$ or $(d_1, d_2, 4^2, 3^2, 2^t, 1^{n-6-t})$ implies $d_1 + d_2 \leq n + t + 2$;
- (7) $\pi = (d_1, d_2, 4, 3^4, 2^t, 1^{n-7-t})$ implies $d_1 + d_2 \leq n + t + 3$;
- (8) For $t = 5, 6$, $\pi \neq (n-i, k+i, 4^t, 2^{k-t}, 1^{n-2-k})$ where $i = 1, \dots, \lfloor \frac{n-k}{2} \rfloor$ and $k = t, \dots, n-2i$;
- (9) $\pi \neq (5^4, 3^2, 2), (4^6), (3^6, 2), (6^4, 3^4), (4^2, 3^6), (4, 3^6, 2), (3^6, 2^2), (3^8), (3^7, 1), (4, 3^8), (4, 3^7, 1), (3^8, 2), (3^7, 2, 1), (3^9, 1), (3^8, 1^2), (n-1, 4^2, 3^4, 1^{n-7}), (n-1, 4^2, 3^5, 1^{n-8}), (n-1, 5^3, 3^3, 1^{n-7}), (n-2, 4, 3^5, 1^{n-7}), (n-2, 4, 3^6, 1^{n-8}), (n-3, 3^6, 1^{n-7}), (n-3, 3^7, 1^{n-8})$.

Theorem 42 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 6$. Then π is potentially $K_6 - C_6$ -graphic if and only if the following conditions hold:

- (1) $d_6 \geq 3$;
- (2) For $i = 1, 2$, $d_1 = n - i$ implies $d_{4-i} \geq 4$;

- (3) $d_2 = n - 1$ implies $d_4 \geq 4$;
- (4) $d_1 + d_2 = 2n - i$ and $d_{n-i+3} = 1$ ($3 \leq i \leq n - 4$) implies $d_4 \geq 4$;
- (5) $d_1 + d_2 = 2n - i$ and $d_{n-i+4} = 1$ ($4 \leq i \leq n - 3$) implies $d_3 \geq 4$;
- (6) $\pi = (d_1, d_2, d_3, 3^k, 2^t, 1^{n-3-k-t})$ implies $d_1 + d_2 + d_3 \leq n + 2k + t + 1$;
- (7) $\pi = (d_1, d_2, 3^4, 2^t, 1^{n-6-t})$ implies $d_1 + d_2 \leq n + t + 2$;
- (8) $\pi \neq (n - i, k, t, 3^t, 2^{k-i-t-1}, 1^{n-2-k+i})$ where $i = 1, \dots, \lfloor \frac{n-t-1}{2} \rfloor$ and $k = i + t + 1, \dots, n - i$ and $t = 4, 5, \dots, k - i - 1$;
- (9) $\pi \neq (3^6, 2), (4^2, 3^6), (4, 3^6, 2), (3^6, 2^2), (3^8), (3^7, 1), (4, 3^8), (4, 3^7, 1), (3^8, 2), (3^7, 2, 1), (3^9, 1), (3^8, 1^2), (n - 1, 4^2, 3^4, 1^{n-7}), (n - 1, 4^2, 3^5, 1^{n-8}), (n - 2, 4, 3^5, 1^{n-7}), (n - 2, 4, 3^6, 1^{n-8}), (n - 3, 3^6, 1^{n-7}), (n - 3, 3^7, 1^{n-8})$.

Xu and Hu[57] characterized the potentially $K_{1,4} + e$ -graphic sequences. Chen and Li [8] characterized the potentially $K_{1,t} + e$ -graphic sequences.

Theorem 43 Let $\pi = (d_1, d_2, \dots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_{1,4} + e$ -graphic if and only if $d_1 \geq 4, d_3 \geq 2$.

Theorem 44 Let $t \geq 3$, $\pi = (d_1, d_2, \dots, d_n)$ is a graphic sequence with $n \geq t + 1$. Then π is potentially $K_{1,t} + e$ -graphic if and only if $d_1 \geq t, d_3 \geq 2$.

Open Problems

Problem 1. Determine $\sigma(K_{r+1} - G, n)$ for G is a graph on k vertices and j edges which contains a graph $K_3 \cup K_{1,3}$ but not contains a cycle on 4 vertices and not contains Z_4, P_3 .

Problem 2. Determine $\sigma(K_{r+1} - G, n)$ for $G = K_3 \cup iK_2 \cup jP_2 \cup tK_3$.

Problem 3. Determine $\sigma(K_{r+1} - G, n)$ for graph G which contains C_3, C_4, \dots, C_l but not contains a cycle on $l + 1$ vertices ($4 \leq l \leq r$).

Problem 4. Determine $\sigma(K_{r+1} - G, n)$ for graph G which contains C_3, C_4, \dots, C_{r+1} .

Problem 5. Determine $\sigma(K_{r+1} - G, n)$ for small n .

Problem 6. Characterize potentially $K_{r+1} - G$ -graphic sequences for the remaining G .

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